Multiple Preference Representation Formats in DM: An overview of their integration and applications

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Outline

Decisions and Preferences

Decisions depend, at least in part, on preferences

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DEMOCRATIC THEORY *is based on the premise that the resolution of a matter of social policy, group choice or collective action should be based on the desires or preferences of the individuals in the society, group or collective*

Peter C. Fishburn (1973)
The Theory of Social Choice
New Jersey: Princeton University Press

Decision Problem

- X finite set of alternatives
- E finite set of experts
- Each expert provides preferences over X

Aim: Derive a collective preference ordering of alternatives
Choice function/priority vector

Multiple Preference Representation Formats

It is argued that

- Unless we force the experts in the group
- Quite natural to expect cases where experts making group decision could provide their preferences in different ways
- Each expert is characterised by their own ideas, attitudes, motivation and personality

Multiple Preference Representation Formats

Up to 1995:
Aforementioned argument

- Backed up by research literature – many papers published different GDM problems with preferences represented homogeneously using different preference representation formats
- No paper dealt with heterogeneous preferences
- Some papers used transformations from one format to another one, mainly the so-called fuzzy preference relation
- Use of appropriate tools developed for the transformed representation format

Multiple Preference Representation Formats

Up to 1995:
Gap on the theory of group decision making

- Lack of sound mathematical models to integrate multiple preference representation formats
- Aim of the research conducted by Francisco Chiclana (numerical information) and Luis Martínez (numerical/linguistic information) under the supervision of Prof. Herrera and Prof. Herrera-Viedma

Multiple Preference Representation Formats

Good starting point of research:
Tanino's paper published in 1984 in FSS[1]

- Expert's preferences are viewed as representing intensity of preference via the concept of a reciprocal fuzzy preference relation
- 'Individual preferences are given by some utility functions or utility values'
- Proposes transformation functions
  - to derive fuzzy preference relations from a utility function on a set of alternatives
  - given without any mathematically sound results to justify their validity

[1] T. Tanino, "Fuzzy preference orderings in group decision making," *Fuzzy Sets Syst.*, vol. 12, pp. 117–131, 1984.

Preference Ordering

- An ordered vector of alternatives from best to worst
- $O = \{o(1), \ldots, o(n)\}$, where $o(\cdot)$ is a permutation function over the index set $\{1, \ldots, n\}$
- Aim achieved
- Good procedures are not available when individuals provide something less than a preference ordering over the set of alternatives[2]

[2] H. Nurmi, "Assumptions of individual preferences in the theory of Voting procedures," in J. Kacprzyk and M. Roubens (Eds.) *Non-Conventional Preference Relations in Decision Making,* Springer Verlag, Berlin, pp. 142–155, 1988.

Utility Function

- A real valued function reflecting the decision maker's preferences
- $U = \{u_i,\ i = 1, \ldots, n\},\ u_i \in [0, 1]$, where u_i represents the utility evaluation given by the expert to the alternative x_i
- Aim achieved

Fuzzy Preference Relation

Bezdek, Spillman and Spillman[3]

Definition (Fuzzy Preference Relation)

A fuzzy preference relation $R = (r_{ij})$ on a finite set of alternatives X is a fuzzy relation in $X \times X$, that is characterized by a membership function $\mu_R : X \times X \longrightarrow [0, 1]$, with the following interpretation:

- $r_{ij} = 1$ indicates the maximum degree of preference for x_i over x_j
- $r_{ij} \in]0.5, 1[$ indicates a definite preference for x_i over x_j
- $r_{ij} = 1/2$ indicates indifference between x_i and x_j
- $r_{ij} + r_{ji} = 1 \ \forall i, j \in \{1, \ldots, n\}$

[3] J. Bezdek, B. Spillman and R. Spillman, "A fuzzy relation space for group decision theory," *Fuzzy Sets Syst.*, vol. 1, pp. 255–268, 1978.

Binary Preference Relations

Subclass of fuzzy preference relations

Given the binary preference relation is preferred to ($\succ$) on a countable set X, with is indifferent to ($\sim$) defined as $x \sim y$ if neither $x \succ y$ nor $y \succ x$, a fundamental result is that there exists a utility function, which is unique up to a positive monotonic transformation,

$$u\colon X \to \mathbb{R}$$

such that

$$x \succ y \Leftrightarrow u(x) > u(y)$$

if and only if $\succ$ on X is a weak order, i.e.,

- $\succ$ is transitive ($x \succ y \wedge y \succ z \Rightarrow x \succ z$)
- $\succ$ is irreflexive (we never have $x \succ x$) and
- $\sim$ is transitive ($x \sim y \wedge y \sim z \Rightarrow x \sim z$)

Fuzzy Preference Relation

Derivation of a preference ordering

- Given a FPR, not necessarily reciprocal, Wang[4] proved that if the following acyclic property was verified

$$\forall i_1, i_2, \ldots, i_m \in \{1, 2, \ldots, n\} : r_{i_1 i_2} > r_{i_2 i_1}, r_{i_2 i_3} > r_{i_3 i_2}, \ldots, r_{i_m i_{m-1}} > r_{i_{m-1} i_m} \Rightarrow r_{i_1 i_m} > r_{i_m i_1}$$

then a total order can be produced in X

- A similar result was obtained by Basile[5] when the FPR is reciprocal and is weakly transitive ($r_{ij} > 0.5 \wedge r_{jk} > 0.5 \Rightarrow r_{ik} > 0.5$)

[4] X. Wang, "An investigation into relations between some transitivity-related concepts," *Fuzzy Sets Syst.,* vol. 89, pp. 257–262, 1997.

[5] L. Basile, "Ranking alternatives by weak transitivity relations." In: J. Kacprzyk and M. Fedrizzi (Eds.) *Multiperson Decision Making using Fuzzy Sets and Possibility Theory,* Kluwer Academic Publishers, Dordrecht, pp. 105 –112, 1990.

Fuzzy Preference Relation

Orlovski's non-dominance criterion

- Orlovski[6] proposed a rational criterion to produce a total order on X based on the strict preference relation

$$R^s = (r_{ij}^s) \quad \text{with} \quad r_{ij}^s = \max\{r_{ij} - r_{ji}, 0\}$$

  and the concept of non-dominance.
- A quantifier non-dominance degree that extended Orlovski's non-dominance degree was proposed by Chiclana et al.[7]

[6] S.A. Orlovski, "Decision-making with fuzzy preference relations," *Fuzzy Sets Syst.*, vol. 1, pp. 155–167, 1978.

[7] F. Chiclana, F. Herrera, E. Herrera-Viedma and M.C. Poyatos, "A classification method of alternatives for preference ordering criteria based on fuzzy majority," *The Journal of Fuzzy Mathematics* vol. 4, no. 4, pp. 801–813, 1996.

Multiplicative Preference Relation

Saaty[8]

Definition (Multiplicative Preference Relation)

A reciprocal preference relation A on a finite set of alternatives X is is represented by a matrix $A = (a_{ij})$, being a_{ij} interpreted as x_i is a_{ij} times as good as x_j, based on the 1 to 9 ratio–scale:

- $a_{ij} = 1$ indicates indifference between x_i and x_j
- $a_{ij} = 9$ indicates that x_i is absolutely preferred to x_j
- $a_{ij} \in \{2, \ldots, 8\}$ indicates intermediate preference evaluations
- $a_{ij} \cdot a_{ji} = 1 \; \forall i, j \in \{1, \ldots, n\}$.

[8] Th.L. Saaty, "Exploring the interface between hierarchies, multiple objectives and fuzzy sets," *Fuzzy Sets Syst.*, vol. 1, pp. 57–68, 1978.

Multiplicative Preference Relation

Priority vector

- Saaty proposed the eigenvector method to obtain a vector of priorities
- This method is designed under the condition of multiplicative consistency
- Under this consistency property, Saaty proves that there exists a set of priorities (utilities)

$$\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$$

  such that $a_{ij} = \frac{\lambda_i}{\lambda_j}$

- This set of values is unique up to positive linear transformation $f(\lambda_i) = \beta \cdot \lambda_i$ with $\beta > 0$

Linguistic Preference Relation

Definition (Linguistic Preference Relation)

A linguistic preference relation L on a finite set of alternatives X is characterized by a membership function $\mu_L : X \times X \longrightarrow S$ where $S = \{s_0, s_1, \ldots, s_g\}$ is a linguistic term set, $\#(S) = g + 1$. The following properties are imposed to S:

1. The set is ordered: $s_i \geq s_j$, if $i \geq$ j.
2. There is the negation operator: $Neg(s_i) = s_j$ such that $j = g - i$.
3. There is the min operator: $Min(s_i, s_j) = s_i$ if $s_i \leq s_j$.
4. There is the max operator: $Max(s_i, s_j) = s_i$ if $s_i \geq s_j$.

Linguistic Preference Relation

Rank ordering of alternatives

- Herrera and Herrera-Viedma[9] propose a consistent linguistic choice mechanisms that acts on the transitive linguistic preference relation CC(L) of L
- Application of a conjunction-sequential linguistic choice mechanism
- Based on four linguistic choice sets of alternatives
  - of greatest alternatives
  - of nondominated alternatives
  - of strictly greatest alternative
  - of maximal alternatives

[9] F. Herrera and E. Herrera-Viedma, “Choice functions and mechanisms for linguistic preference relations ,” *Eur. J. Oper. Res.*, vol. 1120, pp. 144–161, 2000.

Tanino's work

It is worth mention his assertion

Since fuzzy theory in itself has aspect quite similar to probability theory, fuzzy preference orderings considered in the following naturally have some analogies in concepts with the traditional theory of probabilistic choice

In probabilistic choice theory

- Binary preference theories attempt to describe the probability structure of all subsets of two-alternatives of X
- $p(x, y)$ means probability that x is chosen when $\{x, y\}$ is presented
- Fuzzy preference relations describe the binary preference probabilities of all subsets of two-alternatives of X, known as 'probabilistic binary preference relations'

In probabilistic choice theory

- In[10] three constant utility models are defined
  1. weak
  2. strong
  3. strict
- The utility function is a fixed numerical function over the outcomes

[10] R.D. Luce and P. Suppes, "Preferences, utility and subject probability." in R.D. Luce, R.R. Bush and E. Galanteret (Eds.) *Handbook of Mathematical Psychology*, Vol. III, Wiley, New York, pp. 249–410, 1965.

Constant Utility Models

Weak

A weak binary utility model is a set of binary preference probabilities for which there exists a real–valued function w over X such that

$$p(x,y) \geq \frac{1}{2} \text{ if and only if } w(x) \geq w(y), \quad x, y \in X$$

w must be on an ordinal scale, i.e. w is unique up to strictly monotonic increasing transformation

Constant Utility Models

Strong

A strong binary utility model is a set of binary preference probabilities for which there exist a real–valued function u over X and a cumulative function ϕ such that
$\forall x, y \in X$ for which $p(x, y) \notin \{0, 1\}$,

$$p(x, y) = \phi[u(x) - u(y)]$$

with

$$\phi(0) = \frac{1}{2}$$

u is an interval scale, i.e. u can be transformed by positive linear transformations

Constant Utility Models

Strict

A strict binary utility model is a set of binary preference probabilities for which there exists a positive real–valued function v over X such that $\forall x, y \in X$ for which $p(x, y) \notin \{0, 1\}$,

$$p(x, y) = \frac{v(x)}{v(x) + v(y)}$$

v is a ratio scale, i.e. v is determined up to multiplication by a positive constant

Tanino's work

Assumption

Based on his viewpoint about fuzzy preferences, which is:

> $r_{ij} > \frac{1}{2}$ *indicates a definite preference of* x_i *to* x_j*, the intensity of preference varying from 'mild' to 'strong'*

Tanino assumes that:

> *individual preferences are given by some utility function or utility values*

Tanino's work

Proposal: **Utility values can be identified with transitive fuzzy preference relations**

- Utility function u on X given as a difference scale

$$r_{ij} = \frac{1}{2}[1 + u(x_i) - u(x_j)]$$

  Fuzzy preference relation satisfies additive transitivity
- Utility function v on X given as a positive ratio scale

$$r_{ij} = \frac{v(x_i)}{v(x_i) + v(x_j)}$$

  Fuzzy preference relation satisfies multiplicative transitivity
- Conversely, from a transitive (additive/multiplicative) fuzzy preference relation alternatives can be assigned unique utility values (up to addition of/multiplication by a positive scalar)

Extending Tanino's Proposals

Utility Values and Fuzzy Preference Relations – Ratio Scale[11]

Proposition

Let $U = \{v_1,\ v_2, \ldots,\ v_n\}$ be a set of ratio scale evaluations associated to a set of alternatives X. The intensity of preference of alternative x_i over alternative x_j, r_{ij}, is given by the following transformation function

$$r_{ij} = \begin{cases} \dfrac{s(v_i)}{s(v_i) + s(v_j)} & \text{if } (v_i, v_j) \neq (0,0) \\ \dfrac{1}{2} & \text{if } (v_i, v_j) = (0,0) \end{cases}$$

with $s\colon [0,1] \to \mathbb{R}^+$ a non decreasing and continuous function, verifying $s(z) = 0$ iff $z = 0$.

When $s(z) = z$ we obtain Tanino's proposal.

[11] F. Chiclana, F. Herrera and E. Herrera-Viedma, "Integrating three representation models in fuzzy multipurpose decision making based on fuzzy preference relations," *Fuzzy Sets Syst.*, vol. 97, no. 1, pp. 33–48.

Extending Tanino's Proposals

Utility Values and Fuzzy Preference Relations – Difference Scale[12]

Proposition

Let $U = \{u_1, \ u_2, \ldots, \ u_n\}$ be a set of difference scale evaluations associated to a set of alternatives X. The intensity of preference of alternative x_i over alternative x_j, r_{ij}, is given by the following transformation function

$$r_{ij} = \frac{1}{2}[1 + F(u_i - u_j) - F(u_j - u_i)]$$

with F a non decreasing function, verifying $F(0) = 0$.

When $F(z) = \frac{z}{2}$ we obtain Tanino's proposal.

[12] F. Chiclana, F. Herrera and E. Herrera-Viedma, "Integrating three representation models in fuzzy multipurpose decision making based on fuzzy preference relations," *Fuzzy Sets Syst.*, vol. 97, no. 1, pp. 33–48, 1998.

Preference Ordering $\longmapsto$ Fuzzy Preference Relation

$O = \{o(1), \ldots, o(n)\} \longmapsto R = (r_{ij})$

1. Each position is associated a utility value via a non-decreasing function
$$u_i = u(n - o(i))$$
2. Utility values are considered to be given on a difference scale
3. Example of such function is
$$u(n - o(i)) = \frac{n - o(i)}{n - 1}$$
4. Apply previous result taking $F(z) = \frac{z}{2}$
$$r_{ij} = \frac{1}{2}\left[1 + \frac{o(j)}{n-1} - \frac{o(i)}{n-1}\right]$$

Multiplicative and Fuzzy Preference Relations

MPR $\rightarrow$ FPR[13]

Proposition

Let X be a set of alternatives, and associated with it a multiplicative preference relation $A = \left(a_{ij}\right)$. Then, the corresponding additive fuzzy preference relation, $R = \left(r_{ij}\right)$, associated with A is given as follows:

$$r_{ij} = g\left(a_{ij}\right) = \frac{1}{2}\left(1 + \log_9 a_{ij}\right)$$

[13] M. Fedrizzi, "On a consensus measure in a group MCDM problem," in J. Kacprzyk and M. Fedrizzi (Eds.) *Multiperson Decision Making Models using Fuzzy Sets and Possibility Theory,* Kluwer Academic Publishers, Dordrecht, pp. 231 – 241, 1990.
F. Chiclana, F. Herrera and E. Herrera-Viedma, "Integrating multiplicative preference relations in a multipurpose decision-making model based on fuzzy preference relations," *Fuzzy Sets Syst.*, vol. 122, no. 2, pp. 277–291, 2001

Combining Numeric and Linguistic Information

First approach based on characteristic values[14]

Given a linguistic label $s_i \in S$, a set of characteristic values (maximum value, centre of gravity, ...) is considered

$$CF_i = \{G_1(s_i), G_2(s_i), \ldots, G_z(s_i)\}$$

L→N $\psi^N(s_i) = g(G_1(s_i), G_2(s_i), \ldots, G_z(s_i))$
g being an aggregation operator

N→L $\psi^L(r) = s_i$
s_i being the label with $r \in Supp(s_i)$ and minimum aggregated distance from r to its characteristics values

[14] M. Delgado, F. Herrera, E. Herrera-Viedma, and L. Martínez, "Combining numerical and linguistic information in group decision making," *Information Sciences*, vol. 107, pp. 177–194, 1998.

Combining Numeric and Linguistic Information

Second approach[15]

1. Convert a numerical value v into a fuzzy set on S

$$v \longmapsto \tau(v) = \{(s_i, \mu_{s_i}(v))/s_i \in S\}$$

2. Transform above fuzzy set into a linguistic 2-tuple assessed on S

$$\tau(v) \longmapsto (s_k, \alpha)$$

$$k = round\left(\frac{\sum_{j=0}^{g} j \cdot \mu_{s_j}(v)}{\sum_{j=0}^{g} j}\right)$$

$$\alpha = \frac{\sum_{j=0}^{g} j \cdot \mu_{s_j}(v)}{\sum_{j=0}^{g} j} - k$$

[15] F. Herrera and L. Martínez, "An approach for combining linguistic and numerical information based on the 2-tuple fuzzy linguistic representation model in group decision making," *Int. J. Uncertainty, Fuzziness and Knowledge-Based Systems*, vol. 8, pp. 539–562, 2000.

Multigranular Information

LTSs of different cardinality used to express preferences on set of alternatives[16]

1. Basic linguistic term set S_T
   1. Only one linguistic term set with maximum granularity: choose it
   2. Two or more linguistic term sets with maximum granularity, then
      1. Same semantics, with different labels, choose any one of them
      2. Different semantics, define generic linguistic term set with granularity greater than the number of terms a person is able to discriminate
2. $\tau_{AS_T} : A \longrightarrow F(S_T)$

$$\tau_{AS_T}(l_i) = \{(c_k, \alpha_k^i) \,/ k \in \{0, \ldots, g\}, \ \forall l_i \in A$$

$$\alpha_k^i = \max_y min\{\mu_{l_i}(y), \mu_{c_k}(y)\}$$

Extends previous numeric–linguistic second approach

[16] F. Herrera, E. Herrera-Viedma and L. Martínez, "A fusion approach for managing multi-granularity linguistic term sets in decision making", *Fuzzy Sets and Syst.*, 114, pp. 43–58, 2000.

Alternative Integration Approach

Optimization

- In 2004[17], Zhi-Ping Fan and collaborators proposed an optimization approach to integrate FPR and MPR
- In 2006[18], this approach was adapted to include utility values and preference orderings
- Alternative weight vector is defined as the one with corresponding (additive/multiplicative) consistent PR closest to the one given by the expert

[17] Z.-P. Fan, S.-H. Xiao and G.-F. Hu, "An optimization method for integrating two kinds of preference information in group decision-making," *Computers & Industrial Engineering,* 46, pp. 329–335, 2004.

[18] J. Ma, Z.-P. Fan, Y.-P. Jiang and J.-Y. Mao, "An optimization approach to multiperson decision making based on different formats of preference information," *IEEE Transactions on Systems, Man and Cybernetics-Part A: Systems and Humans,* 36(5), pp. 876–889, 2006.

Alternative Integration Approach

Some comments

- Alternative to unification approach
  - "to avoid losing or distorting original preference information"
  - Transformation function between FPR and MPR is bijective!
  - They transform preference orderings into utility values!
  - Consistent PR obtained from weight vector via transformation functions from unification approach!

Alternative Integration Approach

Some comments

- In 2007[19], the previous optimization approach was modified to avoid rank reversal problems
- In 2007[20], a new optimization approach is presented where MPR are integrated with FPR via transformation functions

[19] Y.-M. Wang, Z.-P. Fan and Z. Hua, "A chi-square method for obtaining a priority vector from multiplicative and fuzzy preference relations," *European Journal of Operational Research,* 182, pp. 356–366, 2007.

[20] Y.-M. Wang and Z.-P. Fan, "Group decision analysis based on fuzzy preference relations: logarithmic and geometric least square methods," *Applied Mathematics and Computation,* 194, pp. 108-119, 2007.

Transformation function between FPR and MPR

Bijective Nature

- Xu and Da[21] define the priority vector of FPR to be the transformed priority vector of the equivalent MPR
- Transforming a FPR into a MPR, they propose an optimization problem – least deviation method to derive its priority vector

[21] Z. Xu and Q. Da, "A least deviation method to obtain a priority vector of a fuzzy preference relation," *European Journal of Operational Research,* 164, pp. 206–216, 2005.

Wide Range of Applications

Within Industry and Academic Sectors

- Selection of lean tools
- Selection of advance manufacturing technology
- Predicting the success of knowledge management implementation
- Measuring Quality of Service, i.e. consumer satisfaction
- Quality Function Deployment
- Cooperative learning in e-learning

Lean Manufacturing

Selection of lean tools[22]

- Shorten the time line between customer order and shipment of goods
- Goal is to reduce 'wastes' in terms of
  - human effort, inventory, time to market and manufacturing space
- To become highly responsive to customer demand
  - most efficient and economic manner

[22] R.K. Singh, S. Kumar, A. K. Choudhury and M. S. Tiwari, "Lean tool selection in a die casting unit: a fuzzy;based decision support heuristic" *International Journal of Production Research* 44, pp. 1399–1429, 2006

Selection of lean tools

Authors were part of lean implementation team

- Company designing and manufacturing machine components for automobile industries
- After analysing the problem they implemented
  - A fuzzy AHP process
  - In a particular level DMs were drawn from various section in the company
  - Numerical heterogeneous information on wastes
  - Uniformed using previous transformation function
- Remarkable improvement in 'on-time delivery' and 'machine availability'

Knowledge Management Implementation

Forecasting success/failure[23]

- KM involves innovation and reformation for organizations
- Planning is crucial to ensure successful KM initiative
- Organizations need to be aware of essential factors to implement
- To do that a set of evaluators are asked to provide their opinions

[23] T.-C. Wang and T.-H. Chang, "Application of consistent fuzzy preference relations in predicting the success of knowledge management implementation," *European Journal of Operational Research* 182, pp. 1313–1329, 2007.

Knowledge Management Implementation

Real case study conducted within a LCD manufacturing corporation

- Factors are compared pairwise: only $n-1$ comparisons made
- Also, individual success or failure implementation of such factors are registered
- Linguistic information, which is mapped to Saaty's discrete ratio scale
- Transformed into preference values in $[0, 1]$ via logarithmic function
- Simple averages and normalizations are used
- Weights obtained subsequently used to predict success/failure prediction of their implementation

Summary

- Overview of different methods in literature for integrating different preference representation formats
- Real applications implementing above methods are starting to appear
- However
  - No agreement yet on which transformation functions to use
  - New unification proposals have appeared in literature
  - With little if any evidence supporting their worthiness
  - Specially when dealing with linguistic information

New Unification Proposal

Linguistic Decision Making Model[24]

- Information can be of a numerical or linguistic nature
- Linguistic information is replaced by their corresponding index
- Discrete nature of linguistic information is replaced by a continuous–virtual one
- Numeric to linguistic: it is just a matter of going from $[0, 1]$ to $[-q, q]$
- Multigranular linguistic information: $[-q_1, q_1]$ to $[-q_2, q_2]$
- Uncertainty/vagueness modelled using previous multigranular approach is lost here!

[24] Y. Dong, Y. Xu and S. Yu, "Linguistic multiperson decision making based on the use of multiple preference relations," *Fuzzy Sets and Systems,* 160, pp. 603-623, 2009.